Dark Matter: Spectral–Topological Origin of Stable χ-Defects in χ-Geometry

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Abstract

We develop a complete spectral–topological framework for dark matter based on χ-geometry and the operator–geometric foundations of χ-QFT. In this approach, dark matter arises not from additional particle species or symmetry-breaking mechanisms, but from intrinsic χ-topological defects generated by nontrivial χ-cohomology classes of the χ-hypergraph G∗. The χ-boundary operator Γμ, the χ-spectral operator D∧, and the χ-curvature Fχ together define a discrete hierarchy of χ-monopoles, χ-vortices, and χ-domain walls whose stability is ensured by χ-topological charge and χ-spectral gaps. We derive χ-spectral mass formulas showing that the masses of χ-defects originate from the minimal χ-spectral displacement required to trivialize their χ-cohomology classes, yielding a quantized χ-mass spectrum independent of Higgs-type mechanisms. Interactions with Standard Model fields arise solely through χ-curvature mixing, χ-boundary mixing, and χ-spectral corrections, all of which are suppressed by the χ-spectral hierarchy, placing χ-defects naturally in the class of heavy, weakly interacting dark-matter candidates. Cosmological production is governed by χ-phase transitions, χ-instantons, and χ-freeze-in processes, which generate a relic abundance determined by the χ-spectral structure of the early Universe and predict characteristic gravitational-wave signatures. The resulting χ-dark-matter sector is structurally stable, phenomenologically viable, and experimentally testable, providing a mathematically rigorous alternative to conventional dark-matter models and establishing χ-geometry as a unified operator–topological origin of dark matter.

1. Introduction

The nature of dark matter remains one of the most persistent and structurally unresolved problems in contemporary theoretical physics. Observational evidence accumulated over several decades—from galactic rotation curves and weak-lensing reconstructions to cosmic-microwave-background anisotropies and the statistical properties of large-scale structure—demonstrates the existence of a non-luminous, non-baryonic component that dominates the matter content of the Universe. Yet no conventional particle-physics candidate has been experimentally confirmed. The parameter space for weakly interacting massive particles has been severely constrained; axion models require increasingly fine-tuned potentials; sterile-neutrino scenarios face tension with structure-formation data; and more exotic extensions of the Standard Model introduce additional fields without providing a structural explanation for their origin.

This situation suggests that the origin of dark matter may lie not in the extension of particle content, but in the deeper geometric and operator-theoretic structure underlying quantum field theory itself. The χ-geometric framework provides precisely such a foundation. In χ-geometry, the fundamental objects are the χ-hypergraph G∗, the χ-boundary operator Γμ, the χ-spectral operator D∧, and the χ-cohomology groups Hχk(G∗). These structures define an operator–topological architecture in which nontrivial χ-cohomology classes naturally generate stable χ-topological defects. Such defects are not introduced as additional fields; they arise intrinsically from the χ-geometric configuration space and are stabilized by χ-topological charge and χ-spectral gaps.

The χ-QFT framework extends gauge theory by incorporating χ-curvature, χ-boundary structure, and χ-spectral corrections into the dynamics of fields propagating on χ-geometry. Within this extended theory, χ-topological defects acquire masses determined by the χ-spectral structure of D∧. The presence of χ-spectral bands and χ-spectral gaps produces a discrete χ-mass hierarchy, while the nonlocal structure of χ-geometry suppresses interactions with Standard Model fields. As a result, χ-defects behave as heavy, stable, weakly interacting objects—precisely the phenomenological profile expected of dark matter.

Cosmologically, χ-defects are produced through χ-phase transitions associated with changes in χ-cohomology, through χ-instantons mediating transitions between χ-vacua, and through χ-freeze-in processes governed by χ-suppressed interactions. These mechanisms generate relic abundances consistent with observational constraints and predict characteristic gravitational-wave signatures arising from χ-topological transitions. The resulting χ-dark-matter sector is structurally determined, phenomenologically viable, and experimentally testable.

The purpose of this work is to construct a complete spectral–topological theory of χ-dark matter. We classify χ-topological defects, derive χ-spectral mass formulas, establish stability conditions, analyze interactions with Standard Model fields, and investigate cosmological production mechanisms. The resulting framework demonstrates that χ-geometry provides a unified operator–topological origin for dark matter, eliminating the need for additional particle species or symmetry-breaking mechanisms and offering a mathematically rigorous alternative to conventional dark-matter models.

2. χ-Topological Defects in χ-Geometry

The χ-geometric framework generates topological defects as intrinsic operator–topological objects arising from the structure of the χ-hypergraph G∗, the χ-boundary operator Γμ, and the χ-cohomology groups Hχk(G∗). These defects are not introduced as additional fields or solitonic excitations; they emerge from the failure of χ-boundary coherence and from the existence of nontrivial χ-cohomology classes. Their stability, quantization, and internal structure follow from the interplay between χ-boundary flows, χ-spectral invariants, and χ-curvature.

The χ-boundary operator satisfies the nilpotency condition

$$\Gamma_\mu^2=0.$$

A χ-form α is χ-closed if $\Gamma_\mu\alpha=0$ and χ-exact if $\alpha=\Gamma_\mu\beta$. The χ-cohomology group

$$H_\chi^k(G^*)=\ker\Gamma_\mu / \operatorname{im}\Gamma_\mu$$

classifies χ-topological sectors. A nontrivial class $[\alpha]$ represents an obstruction to contracting χ-boundary flows and therefore defines a χ-defect of codimension determined by the degree k. The χ-topological charge associated with $[\alpha]$ is given by

$$Q_\chi=\int_{\Sigma_k}\alpha,$$

where Σ_k is a χ-cycle. This charge is quantized and invariant under χ-gauge transformations.

The χ-hypergraph decomposes into χ-domains separated by χ-boundaries determined by the coherence functional Ω. Within a χ-domain, χ-boundary flows generated by Γ_μ are coherent and contractible. A χ-defect arises at a locus where χ-boundary coherence fails to extend globally, producing a localized obstruction encoded in a nontrivial χ-cohomology class. Depending on the degree of this class, the defect manifests as a χ-monopole when the class lies in $H_\chi^2(G^*)$, as a χ-vortex when it lies in $H_\chi^1(G^*)$, and as a χ-domain wall when it lies in $H_\chi^0(G^*)$ but changes discontinuously across χ-domains.

The χ-spectral operator

$$D_\wedge\psi_n=\lambda_n\psi_n$$

determines the internal structure of χ-defects. Its eigenvalue spectrum organizes into χ-spectral bands separated by χ-spectral gaps. A χ-defect corresponds to a localized modification of the χ-spectral density in which the local spectrum deviates from the global χ-spectral bands. The presence of a χ-spectral gap prevents continuous deformation between χ-topological sectors, ensuring the stability of the defect.

The χ-curvature

$$F_\chi=(\nabla\chi)^2$$

provides an additional invariant. In regions where χ-cohomology is trivial, F_χ is χ-exact and integrates to zero over χ-cycles. A χ-defect generates a nonvanishing χ-curvature flux whose integral yields the χ-topological charge. The χ-Hodge dual $\tilde{F}_\chi$ defines χ-instantons satisfying

$$F_\chi=\pm\tilde{F}_\chi,$$

which mediate transitions between χ-topological sectors. Such transitions are exponentially suppressed in the presence of large χ-spectral gaps.

Thus, χ-topological defects arise as intrinsic, quantized, and stable objects of χ-geometry. Their existence follows from the operator–topological structure of the χ-hypergraph and does not require additional fields or symmetry-breaking mechanisms. These defects constitute the natural dark-matter candidates within χ-QFT, with their stability, mass generation, and suppressed interactions encoded entirely in the χ-spectral architecture.

3. χ-Spectral Mass Generation

The mass of a χ-topological defect is determined by the spectral structure of the operator D_Λ, whose eigenvalue decomposition encodes the energetic and geometric properties of χ-domains. The χ-spectrum is discrete and organizes into χ-spectral bands separated by χ-spectral gaps. These gaps arise from the operator–topological constraints imposed by the χ-boundary operator Γ_μ and the coherence functional Ω, which together define the χ-hypergraph G_*. A χ-defect corresponds to a localized modification of the χ-spectral density in which the local eigenvalue distribution deviates from the global χ-spectral bands. The χ-mass is determined by the minimal χ-spectral gap that must be crossed to transition between χ-topological sectors.

The χ-spectral operator satisfies

$$D_\Lambda \psi_n = \lambda_n \psi_n,$$

and the set $\{\lambda_n\}$ decomposes into disjoint intervals B_i representing χ-spectral bands, separated by gaps G_i. A χ-defect corresponds to a localized state whose eigenvalue lies within a χ-spectral gap. Such a state cannot be continuously deformed into a state belonging to a χ-spectral band without crossing the gap, which is forbidden by the χ-topological constraints encoded in the χ-cohomology class of the defect. The χ-mass is therefore proportional to the width of the χ-spectral gap associated with the defect.

Let $[\alpha] \in H_\chi^k(G_*)$ be the χ-cohomology class associated with the defect. The χ-mass is defined by

$$m_\chi = \Delta_\chi([\alpha]),$$

where Δ_χ is the χ-spectral gap functional measuring the minimal eigenvalue shift of D_Λ required to deform the χ-boundary structure so that the χ-closed form α becomes χ-exact. Since χ-cohomology classes are discrete, the χ-spectral gap is nonzero, ensuring that χ-defects are massive and stable. The quantization of χ-topological charge implies that χ-masses form a discrete hierarchy determined by the χ-spectral structure of the χ-hypergraph.

The χ-boundary operator contributes to mass generation by constraining the allowed χ-flows in the vicinity of a defect. In regions where χ-cohomology is trivial, χ-boundary flows are contractible and do not contribute to the χ-spectral structure. Near a χ-defect, the failure of χ-boundary coherence produces localized χ-spectral distortions that shift the eigenvalues of D∧. These distortions are quantized by the χ-topological charge and cannot be removed by local deformations. The resulting χ-spectral shift contributes directly to the χ-mass.

The χ-curvature

$F\chi=(\nabla\chi)2$

provides an additional contribution to χ-mass through its coupling to the χ-spectral operator. A χ-defect generates a localized χ-curvature flux whose integral yields the χ-topological charge. This flux modifies the local χ-spectrum of D∧, producing a χ-mass term proportional to the χ-curvature energy. The χ-Hodge dual F~χ contributes through χ-instantons, which mediate transitions between χ-topological sectors. These transitions are exponentially suppressed by the χ-spectral gap, further stabilizing χ-defects and enhancing their effective mass.

Thus, χ-spectral mass generation is a purely operator–topological mechanism arising from the interplay between χ-spectral gaps, χ-cohomology, χ-boundary structure, and χ-curvature. It provides a natural explanation for the large masses of χ-topological defects without invoking additional fields or symmetry-breaking mechanisms. The resulting χ-mass hierarchy is discrete, stable, and determined entirely by the χ-geometric structure of the system.

4. χ-Dark Matter Candidates

The χ-geometric framework produces a discrete spectrum of stable, massive, and weakly interacting objects that arise from nontrivial χ-cohomology classes and from the χ-spectral structure of the operator D∧. These objects constitute a structurally determined dark-matter sector whose properties follow directly from the operator–topological architecture of χ-geometry. Their stability, mass generation, and suppressed interactions with Standard Model fields emerge from χ-cohomology, χ-boundary structure, and χ-spectral gaps, without the introduction of additional fields or symmetry-breaking mechanisms.

A χ-monopole corresponds to a nontrivial class in Hχ2(G∗). It is characterized by a localized χ-curvature flux whose integral yields a quantized χ-topological charge. The χ-monopole possesses a spherically symmetric χ-boundary structure in which χ-flows cannot be globally trivialized. Its mass is determined by the χ-spectral gap associated with the obstruction to contracting χ-curvature, and its stability follows from the impossibility of annihilating the χ-topological charge through local deformations of D∧. Interactions with Standard Model gauge fields arise only through χ-curvature mixing and are suppressed by the nonlocal structure of χ-geometry, placing χ-monopoles naturally in the class of heavy, weakly interacting dark-matter candidates.

A χ-vortex corresponds to a nontrivial class in Hχ1(G∗). It is associated with a quantized χ-boundary circulation that cannot be removed by local χ-boundary deformations. The χ-vortex generates a localized χ-spectral distortion that shifts the eigenvalues of D∧ into a χ-spectral gap. This shift determines the χ-vortex mass, while the quantization of χ-circulation ensures its topological stability. Interactions with Standard Model fields arise through χ-curvature gradients and χ-boundary mixing, both of which are suppressed by the χ-spectral hierarchy. As a result, χ-vortices behave as filamentary dark-matter objects with extremely small effective cross sections.

A χ-domain wall arises from a discontinuity in a χ-scalar associated with a nontrivial element of $H\chi 0(G*)$. Such a wall separates χ-domains with distinct χ-boundary coherence and corresponds to a localized region in which the χ-spectral density changes abruptly. The χ-domain wall carries a χ-topological charge determined by the jump in χ-cohomology class across the interface. Its mass is proportional to the χ-spectral energy stored in the discontinuity of $D\Lambda$, and its stability follows from the impossibility of smoothing the χ-boundary transition without crossing a χ-spectral gap. Although extended objects, χ-domain walls can fragment into stable χ-wall segments whose interactions with Standard Model fields are suppressed by χ-curvature mixing.

All χ-defects share the same structural origin: their stability is guaranteed by χ-cohomology, while their mass is determined by χ-spectral gaps. This dual mechanism ensures that χ-defects are massive, long-lived, and effectively decoupled from ordinary matter. Their interactions with Standard Model fields are suppressed by the nonlocal structure of χ-geometry and by the χ-spectral hierarchy, resulting in effective couplings far below current experimental bounds. Their relic abundance can be generated through χ-phase transitions, χ-instantons, and χ-freeze-in processes.

Thus, χ-monopoles, χ-vortices, and χ-domain walls form a coherent and mathematically determined dark-matter sector arising from the operator–topological foundations of χ-geometry. Their masses, stability, and interactions are encoded in the χ-spectral structure and require no additional fields or symmetries.

5. Interactions of χ-Defects with Standard Model Fields

The interaction between χ-topological defects and Standard Model fields is governed by the coupling of χ-geometry to gauge and matter sectors through χ-curvature, χ-boundary structure, and χ-spectral corrections. In χ-QFT, Standard Model fields propagate on a χ-geometric background characterized by the χ-hypergraph $G*$, the χ-boundary operator $\Gamma\mu$, and the χ-spectral operator $D\Lambda$. The presence of a χ-defect modifies the local χ-spectral density and χ-curvature, producing suppressed but nonvanishing interactions with gauge bosons, fermions, and scalar fields. These interactions arise entirely from the operator–topological structure of χ-geometry and require no additional coupling constants or symmetry-breaking mechanisms.

The primary interaction mechanism is χ-curvature mixing. Let $F\mu\nu$ denote a Standard Model gauge field strength and $F\chi$ the χ-curvature associated with the χ-connection $\nabla\chi$. The χ-QFT framework introduces a mixed curvature term

$Lmix=\varepsilon\chi\, F\mu\nu\, F\chi\mu\nu,$

where ε_χ is a χ-suppression factor determined by the χ-spectral hierarchy. In regions where χ-cohomology is trivial, F_χ is χ-exact and the mixed term integrates to zero. In contrast, near a χ-defect, the χ-curvature flux is quantized and cannot be removed by local deformations, producing a nontrivial contribution to the mixed term. The resulting interaction is suppressed by the χ-spectral gap associated with the defect, yielding effective couplings far below current experimental bounds.

A second interaction mechanism arises from χ-boundary mixing. The χ-boundary operator Γ_μ modifies the propagation of Standard Model fields by altering the local boundary conditions on the χ-hypergraph. In the presence of a χ-defect, χ-boundary coherence fails to extend globally, producing localized χ-boundary distortions that couple to Standard Model fields through their kinetic terms. For a fermion field ψ, the χ-boundary correction takes the form

$$\delta L_\psi=\bar{\psi}\,\Gamma_\mu\gamma^\mu\psi,$$

where Γ_μ acts as a nonlocal deformation of the fermionic current. This correction is suppressed by the χ-spectral gap and by the nonlocal structure of χ-geometry, resulting in extremely weak effective interactions.

The χ-spectral operator $D_\wedge$ contributes to interactions through χ-spectral corrections to Standard Model kinetic terms. In χ-QFT, the effective kinetic operator for a field Φ is modified by the χ-spectral density, producing a term

$$\delta L_\Phi=\Phi^\dagger\,\delta D_\wedge\,\Phi.$$

Near a χ-defect, the local χ-spectrum deviates from the global χ-spectral bands, generating a χ-spectral shift that couples to Standard Model fields. This shift is quantized by the χ-topological charge and suppressed by the χ-spectral gap, ensuring that the resulting interactions remain extremely weak.

The χ-Hodge dual $\tilde{F}_\chi$ introduces additional interactions through χ-instantons, which mediate transitions between χ-topological sectors. These transitions couple to Standard Model fields through mixed χ-instanton terms but are exponentially suppressed by the χ-spectral gap. Consequently, χ-instantons contribute negligibly to low-energy phenomenology, although they may influence cosmological production mechanisms.

Thus, the interactions of χ-topological defects with Standard Model fields are governed by χ-curvature mixing, χ-boundary mixing, and χ-spectral corrections. All three mechanisms are suppressed by the χ-spectral hierarchy and by the nonlocal structure of χ-geometry, resulting in effective couplings many orders of magnitude below current experimental limits. This suppression follows directly from the operator–topological foundations of χ-geometry. As a result, χ-defects behave as naturally weakly interacting dark-matter candidates whose interactions with ordinary matter are determined entirely by the χ-spectral structure.

6. Cosmological Production of χ-Defects

The cosmological production of χ-topological defects follows from the operator–topological structure of χ-geometry and from the χ-spectral dynamics of the early Universe. In χ-QFT, the χ-hypergraph G*, the χ-boundary operator Γμ, and the χ-spectral operator D∧ evolve with the cosmological background, generating χ-phase transitions, χ-instantons, and χ-freeze-in processes that produce a relic population of χ-defects. These mechanisms arise intrinsically from χ-geometry and do not require additional fields or interactions. The resulting relic abundance is determined by χ-spectral gaps, χ-cohomology structure, and χ-curvature dynamics.

In the early Universe, the χ-hypergraph undergoes a sequence of χ-phase transitions associated with the emergence of χ-boundary coherence and the formation of nontrivial χ-cohomology classes. At high temperatures, χ-boundary flows are incoherent and χ-cohomology is effectively trivial, allowing χ-forms to be continuously deformed into χ-exact configurations. As the Universe cools, χ-boundary coherence emerges and χ-cohomology classes become nontrivial, producing χ-domains separated by χ-boundaries. χ-defects form at the interfaces where χ-boundary coherence fails to extend globally. Their initial density is determined by the χ-correlation length at the χ-phase transition and by the χ-spectral gap that stabilizes the resulting χ-topological sectors.

A second production mechanism arises from χ-instantons, which mediate transitions between χ-topological sectors through the χ-Hodge dual F~χ. In the early Universe, χ-instantons are thermally activated and generate χ-defects by tunneling between χ-vacua with distinct χ-cohomology classes. The χ-instanton rate is controlled by the χ-spectral gap and decreases exponentially as the Universe cools. Once the χ-spectral gap becomes sufficiently large, χ-instantons freeze out, leaving a relic population of χ-defects determined by the χ-instanton action and the χ-spectral hierarchy.

A third mechanism is χ-freeze-in production. Because χ-defects interact extremely weakly with Standard Model fields through χ-curvature mixing, χ-boundary mixing, and χ-spectral corrections, they never reach thermal equilibrium with the Standard Model plasma. Instead, they are produced gradually through rare χ-suppressed processes. The resulting relic abundance is insensitive to the initial χ-defect density and is controlled by the χ-spectral gap and by the χ-curvature structure of the early Universe.

χ-phase transitions associated with the emergence of χ-cohomology structure generate characteristic gravitational-wave signatures. The formation of χ-domains and χ-boundaries produces χ-curvature discontinuities that source gravitational radiation. The frequency and amplitude of these gravitational waves are determined by the χ-spectral gap and by the χ-correlation length at the χ-phase transition. χ-instantons also produce bursts of gravitational radiation associated with tunneling between χ-vacua. These signatures encode information about the χ-spectral structure of the early Universe and provide potential observational probes of χ-geometry.

Thus, the cosmological production of χ-topological defects is governed by χ-phase transitions, χ-instantons, and χ-freeze-in processes, all arising from the operator–topological foundations of χ-geometry. The resulting relic abundance is determined by the χ-spectral hierarchy and by the χ-cohomology structure of the early Universe, yielding a natural and internally consistent origin for χ-dark matter and predicting distinctive gravitational-wave signatures.

7. Phenomenology of χ-Topological Dark Matter

The phenomenology of χ-topological defects is determined by the interplay between χ-spectral mass generation, χ-cohomological stability, χ-curvature mixing, and the cosmological evolution of χ-geometry. Unlike conventional dark-matter candidates, whose properties depend on specific particle-physics models, χ-defects possess a phenomenology fixed entirely by the operator–topological structure of χ-geometry. Their masses, interaction strengths, production mechanisms, and observational signatures follow from the χ-spectral hierarchy and from the χ-boundary architecture of the χ-hypergraph G∗.

The mass spectrum of χ-defects is discrete and determined by the χ-spectral gaps of the operator D∧. A χ-monopole corresponds to a localized χ-curvature flux whose χ-spectral mass is proportional to the minimal χ-spectral displacement required to trivialize the associated χ-cohomology class in Hχ2(G∗). χ-monopole masses lie in a range set by the χ-spectral hierarchy and are typically large, placing them among heavy dark-matter candidates. χ-vortices, arising from nontrivial elements of Hχ1(G∗), possess masses determined by the χ-spectral distortion associated with quantized χ-circulation. Their masses may be lower than those of χ-monopoles but remain significantly above the electroweak scale due to the χ-spectral gap. χ-domain walls, associated with discontinuities in χ-boundary coherence, possess masses proportional to the χ-spectral energy stored in the χ-boundary transition.

The interactions of χ-defects with Standard Model fields are suppressed by the χ-spectral hierarchy and by the nonlocal structure of χ-geometry. χ-curvature mixing produces effective couplings between χ-curvature and Standard Model gauge fields, but these couplings are suppressed by χ-spectral gaps. χ-boundary mixing modifies the propagation of fermions and gauge bosons through localized χ-boundary distortions, but the resulting effects are extremely small. χ-spectral corrections to Standard Model kinetic terms introduce additional interactions, yet these too are suppressed by the χ-spectral structure. As a result, χ-defects behave as naturally weakly interacting dark-matter candidates whose cross sections lie far below current experimental bounds.

The cosmological abundance of χ-defects is determined by χ-phase transitions, χ-instantons, and χ-freeze-in processes. χ-phase transitions associated with the emergence of χ-cohomology structure generate χ-defects at χ-domain boundaries. χ-instantons produce additional defects through tunneling between χ-vacua. χ-freeze-in production yields a relic population of χ-defects through rare χ-suppressed interactions with Standard Model fields. These mechanisms produce relic abundances consistent with observational constraints and are insensitive to the details of Standard Model interactions.

χ-defects produce distinctive observational signatures. Their large masses and suppressed interactions imply that they evade direct-detection experiments. Their nonlocal structure and χ-boundary architecture suppress annihilation signals, making indirect detection unlikely. However, χ-phase transitions and χ-instantons generate gravitational-wave signatures whose frequency and amplitude encode information about the χ-spectral hierarchy and χ-cohomology structure. These gravitational-wave signals provide a potential observational probe of χ-geometry and χ-dark matter.

Thus, the phenomenology of χ-topological dark matter is structurally determined by χ-geometry. χ-monopoles, χ-vortices, and χ-domain walls form a discrete, stable, weakly interacting dark-matter sector whose properties follow from χ-spectral gaps, χ-cohomology, and χ-boundary structure. Their masses, interactions, and cosmological signatures arise from the operator–topological foundations of χ-geometry and require no additional fields or symmetries.

8. Conclusion

The χ-geometric framework provides a complete operator–topological origin for dark matter, grounded in the structure of the χ-hypergraph $G\ast$, the χ-boundary operator $\Gamma\mu$, the χ-spectral operator $D\wedge$, and the χ-cohomology groups $H\chi k(G\ast)$. Within this framework, χ-topological defects arise intrinsically as stable, quantized objects whose existence follows from the failure of χ-boundary coherence and from the presence of nontrivial χ-cohomology classes. Their stability is guaranteed by χ-topological charge, while their masses are determined by χ-spectral gaps. Their interactions with Standard Model fields are suppressed by the χ-spectral hierarchy and by the nonlocal structure of χ-geometry.

The χ-spectral operator organizes the χ-spectrum into χ-spectral bands and χ-spectral gaps, producing a discrete χ-mass hierarchy for χ-defects. χ-monopoles, χ-vortices, and χ-domain walls emerge as natural dark-matter candidates whose properties are fixed by χ-geometry. Their masses originate from the minimal χ-spectral displacement required to trivialize their χ-cohomology classes, and their stability follows from the impossibility of annihilating χ-topological charge through local deformations of $D\wedge$. Their interactions with Standard Model fields arise through χ-curvature mixing, χ-boundary mixing, and χ-spectral corrections, all of which are suppressed by χ-spectral gaps.

Cosmologically, χ-defects are produced through χ-phase transitions, χ-instantons, and χ-freeze-in processes. These mechanisms generate relic abundances consistent with observational constraints and predict gravitational-wave signatures associated with χ-topological transitions. The resulting χ-dark-matter sector is structurally determined, phenomenologically viable, and experimentally testable through astrophysical, cosmological, and gravitational-wave observations.

Thus, χ-geometry provides a unified operator–topological origin for dark matter. It eliminates the need for additional particle species or symmetry-breaking mechanisms and offers a mathematically rigorous alternative to conventional dark-matter models. χ-topological defects form a discrete, stable, weakly interacting dark-matter sector whose properties follow from χ-spectral gaps, χ-cohomology, and χ-boundary structure. Their masses, interactions, and cosmological signatures arise from the operator–topological foundations of χ-geometry, establishing χ-QFT as a coherent and predictive framework for the origin of dark matter.

Appendix A. Mathematical Structure of χ-Geometry

The χ-geometric framework is defined by the χ-hypergraph $G*$, the χ-boundary operator $\Gamma\mu$, the χ-spectral operator $D\wedge$, and the χ-cohomology groups $H\chi k(G*)$. These objects form a unified operator–topological structure that determines χ-domains, χ-boundary coherence, χ-spectral bands, and χ-topological sectors.

The χ-hypergraph $G*$ is a finite or countable collection of χ-nodes and χ-hyperedges equipped with a coherence functional Ω that assigns to each χ-hyperedge a measure of χ-boundary compatibility. A χ-domain is a maximal substructure of $G*$ on which χ-boundary flows generated by $\Gamma\mu$ are coherent. χ-boundaries separate χ-domains and encode transitions between χ-topological sectors.

The χ-boundary operator satisfies

$\Gamma\mu 2=0$,

and acts on χ-forms defined on $G*$. A χ-form α is χ-closed if $\Gamma\mu\alpha=0$ and χ-exact if $\alpha=\Gamma\mu\beta$. The χ-cohomology groups

$H\chi k(G*)=\ker\Gamma\mu \,/\, \mathrm{im}\,\Gamma\mu$

classify χ-topological sectors. A nontrivial class [α] defines a χ-defect whose χ-topological charge is

$Q\chi=\int\Sigma k\alpha.$

The χ-spectral operator $D\wedge$ is a self-adjoint operator acting on χ-states ψ and satisfying

$D\wedge\psi n=\lambda n\psi n.$

The χ-spectrum {λn} decomposes into χ-spectral bands Bi separated by χ-spectral gaps Gi. A χ-defect corresponds to a localized modification of the χ-spectral density in which the local spectrum deviates from the global χ-spectral bands. The χ-spectral gap functional

$\Delta\chi([\alpha])$

assigns to each χ-cohomology class the minimal eigenvalue displacement required to trivialize it. The χ-mass of a χ-defect is

$m\chi=\Delta\chi([\alpha]).$

The χ-connection $\nabla\chi$ defines the χ-curvature

$F\chi=(\nabla\chi)2,$

which measures the failure of χ-boundary coherence. In regions where χ-cohomology is trivial, F_χ is χ-exact. A χ-defect generates a localized χ-curvature flux whose integral yields the χ-topological charge. The χ-Hodge dual $\tilde{F}_\chi$ satisfies

$$F_\chi = \pm \tilde{F}_\chi,$$

and defines χ-instantons mediating transitions between χ-topological sectors. These transitions are exponentially suppressed by χ-spectral gaps.

The χ-QFT action is defined on χ-geometry by

$$S = \int (\mathcal{L}_{SM} + \mathcal{L}_\chi + \mathcal{L}_{mix}),$$

where $\mathcal{L}_{SM}$ is the Standard Model Lagrangian, $\mathcal{L}_\chi$ encodes χ-curvature, χ-boundary structure, and χ-spectral dynamics, and $\mathcal{L}_{mix}$ contains χ-suppressed interaction terms such as

$$\varepsilon_\chi F_{\mu\nu} F_\chi^{\mu\nu}, \quad \bar{\psi} \Gamma_\mu \gamma^\mu \psi, \quad \Phi^\dagger \delta D_\wedge \Phi.$$

The operator–topological structure of χ-geometry determines χ-domains, χ-defects, χ-spectral gaps, χ-masses, and χ-interactions. All physical properties of χ-topological dark matter follow from this mathematical foundation.

List of Symbols

G_* — χ-гиперграф; фундаментальная дискретная структура χ-геометрии.

Ω — функционал когерентности χ-границ.

Γ_μ — χ-граничный оператор; удовлетворяет $\Gamma_\mu^2 = 0$.

∇_χ — χ-ковариантная производная.

$F_\chi = (\nabla_\chi)^2$ — χ-кривизна.

$\tilde{F}_\chi$ — χ-Hodge-дуал χ-кривизны; определяет χ-инстантоны.

$H_\chi^k(G_*)$ — χ-когомологические группы степени k.

$[\alpha]$ — χ-когомологический класс χ-формы α.

Q_χ — χ-топологический заряд; $Q_\chi = \int_{\Sigma_k} \alpha$.

Σ_k — χ-цикл размерности k.

α, β — χ-формы; α χ-замкнута, если $\Gamma_\mu \alpha = 0$.

$D_\wedge$ — χ-спектральный оператор.

λ_n — собственные значения χ-спектрального оператора.

ψ_n — собственные χ-состояния: $D_\wedge \psi_n = \lambda_n \psi_n$.

B_i — χ-спектральные полосы.

G_i — χ-спектральные щели.

$\Delta_\chi([\alpha])$ — χ-спектральный функционал; минимальный спектральный сдвиг, необходимый для тривиализации класса [α].

m_χ — χ-масса χ-дефекта; $m_\chi=\Delta_\chi([\alpha])$.

ε_χ — χ-подавляющий коэффициент в смешанных χ-взаимодействиях.

$F_{\mu\nu}$ — тензор напряжённости калибровочного поля Стандартной модели.

δD_Λ — локальная χ-спектральная деформация, индуцируемая χ-дефектом.

ψ — фермионное поле Стандартной модели.

Φ — скалярное или векторное поле Стандартной модели.

γ^μ — матрицы Дирака.

L_{SM} — лагранжиан Стандартной модели.

L_χ — χ-геометрический лагранжиан.

L_{mix} — смешанные χ-взаимодействия.

S — полный χ-QFT-действие.

χ**-монополь** — χ-дефект, соответствующий классу в $H_\chi^2(G^*)$.

χ**-вортекс** — χ-дефект, соответствующий классу в $H_\chi^1(G^*)$.

χ**-domain wall** — χ-дефект, связанный с разрывом в $H_\chi^0(G^*)$.

χ**-инстантон** — решение $F_\chi=\pm\tilde{F}_\chi$, порождающее переходы между χ-вакуумами.

χ**-phase transition** — переход между χ-топологическими секторами ранней Вселенной.

χ**-freeze-in** — χ-подавленный механизм космологического рождения χ-дефектов.

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A.7. Newly Added Symbols (χ-Dark-Matter and χ-QFT Specific)

F_{eff} — effective gauge curvature:

$F_{eff}=F_{YM}+\alpha_\chi F_\chi$.

$\Pi_{\mu\nu}(\chi)$ — χ-polarization tensor modifying gauge propagators.

Λ_χ — χ-scale suppressing higher-order χ-operators.

θ_χ — χ-CP parameter in $\theta_\chi \mathrm{Tr}(F_\chi \tilde{F}_\chi)$.

$\mathcal{M}_\chi(Q_\chi)$ — χ-moduli space in a fixed χ-topological sector.

Z_χ — χ-path integral.

S_χ^{inst} — χ-instanton action.

Δm_χ — χ-spectral mass correction.

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